

## Entry Task: Differentiate

$$1. \quad F(x) = \frac{7}{x^{10}} - 5\sqrt{x^3} + 4\ln(x) = 7x^{-10} - 5x^{3/2} + 4\ln(x)$$

$$2. \quad G(x) = e^{6x} + 5\tan(x) + \pi$$

$$3. \quad H(x) = 2\tan^{-1}(x) - 3 + e$$

$$4. \quad J(x) = x^3 \cos(4x) + \ln(2)$$

$$\boxed{1} \quad F'(x) = 7(-10x^{-11}) - 5\left(\frac{3}{2}x^{1/2}\right) + 4\left(\frac{1}{x}\right) = -70x^{-11} - \frac{15}{2}x^{1/2} + \frac{4}{x}$$

$$\boxed{2} \quad G'(x) = 6e^{6x} + 5\sec^2(x) + 0 = 6e^{6x} + 5\sec^2(x)$$

$$\boxed{3} \quad H'(x) = 2 \frac{1}{1+x^2} - 0 + 0 = \frac{2}{1+x^2}$$

$$\boxed{4} \quad J'(x) = 3x^2 \cos(4x) - 4x^3 \sin(4x) + 0$$

## 4.9 Antiderivatives

**Goal:** Before we jump into defining integrals (ch. 5), we need to remember some derivatives (in reverse).

**Def'n:** If  $g(x) = f'(x)$ , then we say

$g(x) =$  "the derivative of  $f(x)$ ", and

$f(x) =$  "an antiderivative of  $g(x)$ "

**Example:**

Give an antiderivative of  $g(x) = x^2$ .

GUESS:  $G(x) = x^3$       CHECK:  $G'(x) = 3x^2$   
↓ POWER UP ONE      ↑ DOES  
DOESN'T WORK! ←

Now try

$$G(x) = \frac{1}{3} x^3$$

$$\text{CHECK: } G'(x) = \frac{1}{3} 3x^2 = x^2 \quad \checkmark$$

$$G(x) = \frac{1}{3} x^3 \quad \text{IS ONE ANTIDERIVATIVE}$$

BUT, THERE ARE OTHERS

$$\frac{1}{3} x^3 + 14, \quad \frac{1}{3} x^3 + 17, \quad \frac{1}{3} x^3 - 105$$

GENERAL ANTIDERIVATIVE

IS  $G(x) = \frac{1}{3} x^3 + C$  where  $C$  is a constant

## 4.9: LIST OF GENERAL ANTIDERIVATIVES

<b>FUNCTION</b>	<b>ANTIDERIVATIVE</b>
$f(x) = x^n \ (n \neq -1)$	$F(x) = \frac{1}{n+1}x^{n+1} + C$
$f(x) = x^{-1} = \frac{1}{x}$	$F(x) = \ln x  + C$
$f(x) = e^x$	$F(x) = e^x + C$
$f(x) = \cos(x)$	$F(x) = \sin(x) + C$
$f(x) = \sec^2(x)$	$F(x) = \tan(x) + C$
$f(x) = \sec(x)\tan(x)$	$F(x) = \sec(x) + C$
$f(x) = \sin(x)$	$F(x) = -\cos(x) + C$
$f(x) = \csc^2(x)$	$F(x) = -\cot(x) + C$
$f(x) = \csc(x)\cot(x)$	$F(x) = -\csc(x) + C$
$f(x) = \frac{1}{1+x^2}$	$F(x) = \tan^{-1}(x) + C$

Examples (you do):

Find the general antiderivative of

1.  $f(x) = x^6$

2.  $g(x) = \cos(x) + \frac{1}{x} + e^x + \frac{1}{1+x^2}$

3.  $h(x) = \frac{5}{\sqrt{x}} + \sqrt[3]{x^2} = 5x^{-1/2} + x^{2/3}$

4.  $r(x) = \frac{x-3x^2}{x^3} = \frac{x}{x^3} - \frac{3x^2}{x^3} = \frac{1}{x^2} - \frac{3}{x} = x^{-2} - 3 \cdot \frac{1}{x}$

1]  $F(x) = \frac{1}{7} x^7 + C$

2]  $G(x) = \sin(x) + \ln(x) + e^x + \tan^{-1}(x) + C$

3]  $H(x) = 5 \left( \frac{1}{1/2} x^{1/2} \right) + \left( \frac{1}{5/3} x^{5/3} \right) = 10\sqrt{x} + \frac{3}{5} x^{5/3} + C$

4]  $R(x) = \left( \frac{1}{-1} x^{-1} \right) - 3 \ln(x) + C = -\frac{1}{x} - 3 \ln(x) + C$

## Initial Conditions

There is no way to know what "C" is unless we are given additional information about the antiderivative. Such information is called an **initial condition**.

Example:  $f'(x) = e^x + 4x$  and  $f(0) = 5$

Find  $f(x)$ .

**STEP 1**  $F(x) = e^x + 4 \cdot \frac{1}{2}x^2 + C$   
 $F(x) = e^x + 2x^2 + C$

**STEP 2**  $f(0) = 5$   
 $\Rightarrow e^{(0)} + 2(0)^2 + C = 5$   
 $\Rightarrow 1 + 0 + C = 5$   
 $\Rightarrow \boxed{C = 4}$

$f(x) = e^x + 2x^2 + 4$  (Check!!)

Example:  $f''(x) = 15\sqrt{x}$ , and

$$f(1) = 0, f(4) = 1$$

Find  $f(x)$ .

**STEP 1**  $f''(x) = 15x^{1/2}$   
 $\Rightarrow f'(x) = 15 \cdot \frac{2}{3}x^{3/2} + C$   
 $f'(x) = 10x^{3/2} + C$   
 $\uparrow$  unknown constant

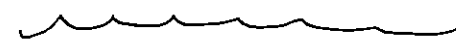
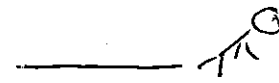
**STEP 2**  $f(x) = 10 \left( \frac{2}{5}x^{5/2} \right) + Cx + D$   
 $f(x) = 4x^{5/2} + Cx + D$

**STEP 3**  $f(1) = 0 \Rightarrow 4(1)^{5/2} + C(1) + D = 0$   
 $\Rightarrow 4 + C + D = 0 \Rightarrow D = -4 - C$   
 $f(4) = 1 \Rightarrow 4(4)^{5/2} + C(4) + D = 1$   
 $128 + 4C + D = 1$

$$128 + 4C + (-4 - C) = 1$$

$3C = -123$   
 $C = -\frac{123}{3} = -41$   
 $D = \cancel{0} 37$

*Motivational:* You know the acceleration or velocity function for some object. What is the original function for the position of the object?



$h(t)$  = height at time  $t$  seconds

$$h(0) = 10$$

$$v(0) = 0 \leftarrow \text{initial velocity "steps off"}$$

$$a(t) = -9.8$$

$$\Rightarrow h''(t) = -9.8$$

$$\Rightarrow h'(t) = -9.8t + C$$

$$\Rightarrow h(t) = -4.9t^2 + Ct + D$$

$$v(0) = 0 \Rightarrow -9.8(0) + C = 0 \Rightarrow C = 0$$

$$h(0) = 10 \Rightarrow -4.9(0)^2 + D = 10 \Rightarrow D = 10$$

$$h(t) = -4.9t^2 + 10$$

*Example:*

Ron *steps off* the 10 meter high dive at his local pool. Find a formula for his height above the water.

(Assume his acceleration is a constant  $9.8 \text{ m/s}^2$  downward)